M2 數學歸納法練習 / M2 mathematical induction

SketchMath 原創中英練習 / Original bilingual practice · Not official HKEAA papers

每題均寫出起始情況、歸納假設與由 k 推至 k+1 的步驟。檢查幾個數值並不是歸納證明。
State the base case, induction hypothesis and k-to-k+1 step. Checking a few values is not an induction proof.

1.

以歸納法證明 $1+3+\cdots+(2n-1)=n^2$，$n\geq 1$。
Prove $1+3+\cdots+(2n-1)=n^2$ for $n\geq 1$ by induction.

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2.

以歸納法證明 $1+2+\cdots+n=n(n+1)/2$，$n\geq 1$。
Prove $1+2+\cdots+n=n(n+1)/2$ for $n\geq 1$ by induction.

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3.

以歸納法證明 $2^n\geq n+1$，n 為非負整數。
Prove $2^n\geq n+1$ for nonnegative integers n by induction.

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答案及詳解 / Worked solutions

1.

1. n=1：左邊 $1=1^2$。假設 n=k 時總和為 k^2。
   Base n=1: $1=1^2$. Assume the sum is k^2 for n=k.
2. n=k+1：總和=$k^2+(2k+1)=(k+1)^2$。
   At n=k+1: sum=$k^2+(2k+1)=(k+1)^2$.
3. 起始及歸納步均成立，故對所有整數 n≥1 成立。
   The base and induction step hold, so the statement holds for all integers n≥1.

證畢。
Proved.

2.

1. n=1：1=1(2)/2。假設總和為 k(k+1)/2。
   At n=1: 1=1(2)/2. Assume the sum is k(k+1)/2.
2. 加 k+1，得 (k+1)(k+2)/2。
   Adding k+1 gives (k+1)(k+2)/2.
3. 因此對所有整數 n≥1 成立。
   Therefore it holds for all integers n≥1.

證畢。
Proved.

3.

1. n=0：1≥1。假設 $2^k \geq k+1$。
   At n=0: 1≥1. Assume $2^k \geq k+1$.
2. 2^(k+1)≥2k+2≥k+2，因 k≥0。
   2^(k+1)≥2k+2≥k+2 because k≥0.
3. 故歸納步成立。
   The induction step follows.

對所有整數 n≥0 成立。
True for every integer n≥0.