M1 抽樣與置信區間練習 / M1 sampling and confidence intervals

SketchMath 原創中英練習 / Original bilingual practice · Not official HKEAA papers

以下使用已知總體標準差的平均數常態模型；不要把總體標準差與樣本平均的標準誤混淆。
Use a normal sampling model with known population standard deviation. Distinguish population standard deviation from the standard error of the sample mean.

1.

獨立同分佈樣本來自標準差 12 的正態總體，n=36。求樣本平均的標準誤。
An independent identically distributed sample of size 36 is drawn from a normal population with standard deviation 12. Find the standard error of its mean.

_____

_____

2.

正態總體的已知 σ=10，獨立樣本 n=100，樣本平均 50。用 z=1.96 建立 95% 平均數置信區間。
A normal population has known σ=10. An independent sample has n=100 and mean 50. Use z=1.96 for a 95% confidence interval for the population mean.

_____

_____

3.

在 σ 與置信水平不變下，要把區間寬度減半，樣本量應如何改變？
With σ and confidence level unchanged, how must sample size change to halve the interval width?

_____

_____

答案及詳解 / Worked solutions

1.

1. SE = 12/√36。
   SE = 12/√36.
2. 不要除以 36。
   Do not divide by 36.

2。
2.

2.

1. SE = 10/√100 = 1；誤差界限 = 1.96。
   SE = 10/√100 = 1; margin of error = 1.96.
2. 下限 50 − 1.96；上限 50 + 1.96。
   Lower bound 50 − 1.96; upper bound 50 + 1.96.

[48.04, 51.96]。
[48.04, 51.96].

3.

1. 若新樣本量為 cn，寬度變成原來的 1/√c。
   If the new size is cn, width becomes 1/√c of the original.
2. 1/√c = 1/2，所以 c = 4。
   1/√c = 1/2, so c = 4.

樣本量乘 4。
Multiply sample size by 4.