M1 求導、積分與梯形法則練習 / M1 differentiation, integration and trapezoidal rule

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分清定積分與總面積；梯形法則一般是近似值。
Distinguish a definite integral from total area; the trapezoidal rule usually gives an approximation.

1.

對 $y=3x^2-4x+1$ 求導，並求駐點的 x 坐標。
Differentiate $y=3x^2-4x+1$ and find the stationary x-coordinate.

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2.

計算 $\int_0^2(3x^2-4x+1)\,dx$。
Evaluate $\int_0^2(3x^2-4x+1)\,dx$.

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3.

以兩個等闊區間，用梯形法則近似 $\int_0^2 x^2\,dx$。
Approximate $\int_0^2 x^2\,dx$ using the trapezoidal rule with two equal subintervals.

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答案及詳解 / Worked solutions

1.

1. dy/dx = 6x − 4。
   dy/dx = 6x − 4.

2. 6x − 4 = 0，得 x = 2/3；二階導數為 6，所以是極小。
   6x − 4 = 0 gives x = 2/3; the second derivative is 6, so it is a minimum.

dy/dx=6x−4；x=2/3。
dy/dx=6x−4; x=2/3.

2.

1. 原函數為 $x^3 - 2x^2 + x$。
   An antiderivative is $x^3 - 2x^2 + x$.

2. 代入 2 和 0：8 − 8 + 2 = 2。這是帶符號的積分，不是總面積。
   At 2 and 0: 8 − 8 + 2 = 2. This is a signed integral, not total area.

2。
2.

3.

1. T = (1/2)[0 + 2(1) + 4] = 3。
   T = (1/2)[0 + 2(1) + 4] = 3.

2. 精確值為 8/3；曲線向上凹，所以此近似高估。
   The exact value is 8/3; the curve is convex, so this approximation overestimates.

近似值 3。
Approximation 3.